3 Chapter 3 The APL Program "System"

This chapter is a collection of program modules, called functions in APL. Together they compromise a program set know as SYSTEM. These modules are the ones referenced throughout Chapter 2.

The serious student of APL will wish to study these in depth. The reader who is only interested in their application to circuit design may skip this chapter.

These modules have been developed by Dr. Svoboda over the past several years and are successfully used in his popular Logic Circuits Laboratory course at the University of California, Los Angeles.[1] The user, either student or scientist, needs only a rudimentary introduction to APL to be able to apply SYSTEM to design problems.

System is used to transform problem specifications into a set of Boolean functions defined by a truth table. It is the basic starting program for study as it will provide the Existence Function of the system of Boolean functions and is the program which provides for minimization and logical relational analysis.

```
     ∇ R←DECIDONT F
[1]    R←¯1+(F=2)/⍳⍴F
     ∇

     ∇ R←DECIMIN F
[1]    R←¯1+(F=1)/⍳⍴F
     ∇

     ∇ R←K BUILDIF F;A;B;C;G;D;H
[1]      →(B≠⌊B←2⊛A←0.5×A←⍴F←2-F)/0
[2]      D←⍉(2,A)⍴2×⍳A
[3]      C←⍳B←B+1
[4]      C[K,B]←B,K
[5]      G←C NEWORDER G←A⍋G←C NEWORDER F
[6]      G←G×F
[7]      R←(G=2)+2×G=0
[8]      H←,⍉((2*K-1),2*1+B-K)⍴D
[9]      R←(R×R≠2)+H×R=2
[10]     'DONT CARES DESIGNATED WITH EQUAL NUMBERS'
[11]     '(LARGER THAN ONE) MUST BE GIVEN EQUAL TRUTH-VALUES.
     ∇

     ∇ YX←F CHART WIDE;XH;XV;MX;MY
[1]    XV←(⍴F)⍴WIDE
[2]    XH←WIDE
[3]    YX←(XV,XH)⍴F
[4]    MX←((0=2|XH)×(2⊛XH))+(0=3|XH)×(3⊛XH)
[5]    MY←(⍴F)⍴MX
[6]    'HORIZONTAL SCALE: (X J) FOR  J= ';⍳MX
[7]    '  VERTICAL SCALE: REMAINING VARIABLES.'
     ∇
```

[1] This lab is no longer available; Svoboda retired in the 1980s and died shortly thereafter.

```
     ∇ R←COMB N;BMB;P;U;CMB
[1]    BMB←R←2*(¯1+⍳N)
[2]    P←1
[3]   H1:CMB←⍳0
[4]    U←P
[5]   H2:CMB←CMB,((BMB<2*U)/BMB)+2*U
[6]    →(N>U←U+1)/H2
[7]    R←R,CMB
[8]    BMB←CMB
[9]    →(N>P←P+1)/H1
     ∇

     ∇ DEGENERATION F;NX;NNX;L1;F0;F1;F2;F3;F4;F5;F6;F7;F8;
continue on the same line..  LIM;U;R;MX;F9;V;K
[1]    F3←'ABCDEFGHJKLMN'
[2]    →(0=F0←+/(F=1))/0
[3]    L1←F0←⌈2⊛F0
[4]    →(F0<NX←2⊛NNX←⍴F)/D1
[5]    'NO DEGEN.'
[6]    →0
[7]   D1:L1←NX-F0
[8]    F5←⍳NX
[9]    F1←COMB NX
[10]   F2←NX⍴2
[11]   LIM←⍴F1
[12]  D7:U←1
[13]  D2:→(L1=+/1=F4←F2⍳F1[U])/D3


[14]  D8:→(LIM≥U←U+1)/D2
[15]   →D4
[16]  D3:F6←F4/F5
[17]   F7←((~F5∈F6)/F5),(F5∈F6)/F5
[18]   R←F7 NEWORDER(F=1)
[19]   MX←((2*⍴F6),2*NX-⍴F6)⍴R
[20]   F8←+⌿MX
[21]   →(0=+/F8>1)/D5
[22]   →D8
[23]  D5:'VARIABLES (X ';F6;') ARE FUNCTIONS OF ALL
                REMAINING VARIABLES'
[24]   'DISCRIMINANT'
[25]   MX
[26]   V←K←1
[27]  D6:F9←NNX⍴(V⍴0),V⍴1
[28]   F9←⊖((2*NX-L1),2*L1)⍴F9
[29]   F9←MX×F9
[30]   F9←+⌿F9
[31]   '(X ';F6[K];') → ';F9
[32]   V←2×V
[33]   →(L1≥K←K+1)/D6
[34]   →D8
[35]  D4:→(0<L1←L1-1)/D7
[36]   →0
     ∇

     ∇ DISCRIMINANT
[1]    ⍝ EXISTENCE FUNCTION CHART
[2]    F CHART NNX
[3]    R←(NX,(0.5×NNX))⍴0
     ∇

     ∇ EQUATION
[1]    ⍝ CONSTRAINT INSERTION
[2]    'WRITE THE EQUATION IN THE PRESCRIBED FORM:'
[3]    'F←F∧(ANY LOGICAL RELATION)'
[4]    'EXECUTE IT AND CALL:  EQUATION, DISCRIMINANT, SOLVE'
    ∇
```



```
    ∇ FFALSE
[1]   TS←CM←5
[2]   ' WRITE A SUFFICIENT CONDITION OF  F  IN THE FORM:'
[3]   '      F←',TEX1
[4]   ' EXECUTE IT AND CALL'
[5]   ' EITHER:  FSTOR K  (WHERE K IS THE INDEX OF THE
      FUNCTION  (Y K) )'
[6]   '      OR:  FTRUE'
    ∇


    ∇ FLIST;ONE;DNT
[1]   ⍝ FUNCTION DEFINITION BY LISTING DECIMAL
      EQUIVALENTS
[2]   ' DECIMAL EQUIVALENTS OF ONES (AT LEAST ONE
      ITEM):'
[3]   ONE←,ONE←⍬
[4]   →(0=⍴ONE)/0
[5]   F←(⍳NNX)∈1+ONE
[6]   ' DECIMAL EQUIVALENTS OF DONT CARES:'
[7]   DNT←,DNT←⍬
[8]   F←~(⍳NNX)∈(1+ONE,DNT)
[9]   ' CALL:  FSTOR  K (WHERE K IS WELL SPECIFIED)'
    ∇


    ∇ R←NX FORM TR;TST;V;H;U;LIM
[1]   R←⍳0
[2]   V←1
[3]   LIM←(⍴,TR)÷NX
[4]  P1:H←TR[V;]
[5]   U←1
[6]  P2:→(0=H[U])/P3
[7]   R←R,G1[(2|H[U])+2×U]
[8]  P3:→(NX≥U←U+1)/P2
[9]   →(LIM<V←V+1)/0
[10]  P←R,' + '
[11]  →P1
    ∇


    ∇ FSTOR K
[1]   ⍝ TRUTH TABLE GENERATION
[2]   →(4≠TR←TS)/S1
[3]   FX[K;]←F
[4]   →S2
[5]  S1:→(5≠TR←TS)/S3
[6]   FX[K;]←~F
[7]   →S2
[8]  S3:→(0=+/F×F)/S4
[9]   ' CONTRADICTION, TRY AGAIN BY RECALLING  FTRUE  OR
      FFALSE  (RESP.: FSET )'
[10]  →0
[11] S4:FX[K;]←F+2×~F+F
[12] S2:TR←TS←0
[13]  →(K≥NY)/S5


[14]  ' CALL:  FTRUE,  FFALSE,  FLIST  TO DEFINE THE
      NEXT FUNCTION  (F K)'
[15]  ' WITH  K=';K+1
[16]  →0
[17] S5:' TRUTH TABLE IS READY:'
[18]  'MAY EXECUTE  FX  TO PRINT THE TABLE'
    ∇
```

```
     ∇ FTRUE
[1]    TR←CM←4
[2]    'WRITE A SUFFICIENT CONDITION OF  F  IN THE
       PRESCRIBED FORM:'
[3]    '    F←',TEX1
[4]    'EXECUTE IT AND CALL'
[5]    'EITHER:  FSTOR K  (WHERE  K  IS THE INDEX OF
       THE FUNCTION (Z K) )'
[6]    '     OR:  FFALSE'
     ∇
```

```
     ∇ LOGIC
[1]    G1←' AABBCCDDEEFFGGHHJJKKLLMMNN'
[2]    'NUMBER OF  X←VARIABLES:'
[3]    NNX←2*NX←⎕
[4]    'NX= ';NX
[5]    'SYMBOLS FOR X-VARIABLES: (X J), (X J); J=';⍳NX
[6]    BINX←NX⍴2
[7]    XX←⊖BINX⊤(¯1+⍳NNX)
[8]    'CALL:  TABLE, SPACE'
     ∇
```

```
     ∇ R←MIN I
[1]    R←2-XX[;I+1]
     ∇
```



```
     ∇ MINIMA F;LIM;NX;G;NNX;TRN;S;R;ZE;H;FND;RZZ;NW;U;
       V;IND;NM;RES;RXX;TSP;TST;SUT;TSF;SUF;MEZ;EFF;
       BINZ;DIF;TRY;FSP
[1]    LIM←2+NX←2⊛NNX←⍴F
[2]    AMB←COMB NX
[3]    R←CRI←FND←⍳0
[4]    NZ←+/ZE←F=DIS←0
[5]    S←1
[6]   A0:H←((2*NX-S),2*S)⍴ZE
[7]    FND←FND,,H∨(2*S-1)∘.=H
[8]    →(NX≥S←S+1)/A0
[9]    FND←(NX,NNX)⍴FND
[10]   G←(LIM×F>1)+G+(+⌿FND)×G←F=1
[11]   FND←(2×FND)+⊖(NX⍴2)⊤(¯1+⍳NNX)
[12]   RZZ←2-2|ZE/FND
[13]   FSP←ZE/⍳NNX
[14]  A4:→(0=NW←⌈/(G<LIM)/G)/A8
[15]   NM←+/RES←(G>0)∧G<LIM
[16]   RXX←2-2|RES/FND
[17]   TSP←RES/⍳NNX
[18]  A3:→(0=U←⍴,IND←(G∈NW)/⍳NNX)/A6
[19]  A2:TRY←TRM←(NIM>1)×2-2|NIM←,FND[;IND[U]]
[20]   TST←⍉(NM,NX)⍴TRM
[21]   SUT←,TSP[((+⌿3=RXX+TST)=0)/⍳NM]
[22]   TST←⍉(NZ,NX)⍴TRM
[23]   SUF←,FSP[((+⌿3=RZZ+TST)=0)/⍳NZ]
[24]   →(0=⍴SUF)/A1
[25]   TRN←(2-2|,FND[;1+NNX-IND[U]])×TRM=0
[26]   TST←⍉(NM,NX)⍴TRN
[27]   H←,TSP[((∨⌿3=RXX+TST)=1)/⍳NM]
[28]   SUT←(SUT∈H)/SUT
[29]   →(1=MEZ←2*+/EFF←TRM=0)/A9
[30]   CMB←(AMB<MEZ)/AMB
[31]   BINZ←1+EFF
[32]   S←1
[33]  A7:DIF←BINZ⊤CMB[S]
[34]   TRY←TRM+DIF×2-NIM
[35]   TSF←⍉(NZ,NX)⍴TRY
[36]   →(0=⍴SUF←FSP[((+⌿3=RZZ+TSF)=0)/⍳NZ])/S1
[37]   →(MEZ>S←S+1)/A7
[38]  S1:→(DIS=1)/A5
[39]   MAX←+/TRY>0
[40]   H←⊖2-(NX⍴2)⊤(SUT-1)
[41]   MEZ←2*+/EFF←(TRM=0)∧DIS∨.∧/H=1∘.=H
[42]   →(1=MEZ)/A9
[43]   CMB←(AMB<MEZ)/AMB
[44]   BINZ←1+EFF


[45]   S←1
[46]  S2:DIF←BINZ⊤CMB[S]
[47]   TRY←TRM+DIF×2-NIM
[48]   TST←⍉(NZ,NX)⍴TRY
[49]   →(0=⍴SUF←FSP[((+⌿3=RZZ+TST)=0)/⍳NZ])/S3
[50]   →(MEZ<S←S+1)/S2 ⋆  → A9
[51]  S3:→(MAX=+/TRY>0)/A5
[52]  A9:→(0<U←U-1)/A2
[53]  A6:→(0<NW←NW-1)/A3
[54]   'CYCLE DISSOLUTION'
[55]   DIS←1
[56]   →A4
```

```
[57] A5:→(DIS=0)/A1
[58]  TST←⍉(NM,NX)⍴TRY
[59]  SUT←,TSP[((+⌿3=RXX+TST)=0)/⍳NM]
[60] A1:G[SUT]←LIM
[61]  CRI←CRI,IND[U]-1
[62]  TST←⍉(NM,NX)⍴TRY
[63]  R←R,TRY
[64]  DIS←0
[65]  →A4
[66] A8:S←(⍴(,R))÷NX
[67]  TRI←R←(S,NX)⍴R
[68]  NX FORM R
[69]  'CRITICAL SET: ';CRI
    ∇

    ∇ R←X NEWORDER F;U;B;NX;NNX;BINX
[1]   →(NX≠⌊NX←2⊛NNX←⍴F)/0
[2]   →(NX≠⍴X)/0
[3]   BINX←NX⍴2
[4]   R←F
[5]   U←0
[6]  N2:B←⍉BINX⊤U
[7]   R[1+2⊥⍉B[X]]←F[U+1]
[8]   →(NNX>U←U+1)/N2
[9]   →0
    ∇

    ∇ PRIMIMPLICANT OF;A;U;V;S;RR;IMP;DIS;RS;TRTX;PP;NX
[1]   NX←2⊛⍴OF
[2]   'ALGEBRAIC FORM OF THE COMPLEMENTARY FUNCTION:'
[3]   MINIMA(OF=0)
[4]   RR←TRI
[5]   S←(⍴RR)[1]
[6]   TRTX←NX⍴3
[7]   IMP←¯1+⍳3*NX
[8]   PP←⍳0
[9]  P3:DIS←0,⍳0
[10]  RS←,RR[S;]
[11]  U←1
[12] P2:→(RS[U]=0)/P1
[13]  DIS←DIS,(DIS+RS[U]×3*U-1)
[14] P6:→(NX≥U←U+1)/P2
[15]  IMP←(~(IMP∈DIS))/IMP
[16]  →(0<S←S-1)/P3
[17]  →P7
[18] P1:DIS←DIS,(DIS+3*U-1),(DIS+2×3*U-1)
[19]  →P6
[20] P7:→(0=⍴IMP)/P8
[21]  PP←PP,A←⌊/IMP
[22]  RS←TRTX⊤A
[23]  DIS←A,⍳0
[24]  V←NX
[25] P5:→(RS[V]>0)/P4
[26]  DIS←DIS,(DIS+3*NX-V),(DIS+2×3*NX-V)
[27] P4:→(0<V←V-1)/P5
[28]  IMP←(~(IMP∈DIS))/IMP
[29]  →P7
[30] P8:PP←⍉⍉TRTX⊤PP
[31]  'SUM OF ALL PRIME IMPLICANTS:'
[32]  NX FORM PP
    ∇
```



```
     ∇ SOLVE;MTX;SET;DON;NIC;VEC;LIM;T;FIX;RES;V;MMR
[1]    MTX←⍉(NNX,NNY)⍴⍳NNY
[2]    FX←(NY,NNX)⍴0
[3]    DSCR←(NNY,NNX)⍴F
[4]    SET←+⌿DSCR
[5]    DON←(NNY=SET)/¯1+⍳NNX
[6]    →(0=⍴DON)/S5
[7]    'EACH SOLUTION HAS THE FOLLOWING DONT CARES: ';DON
[8]   S5:NIC←(0=SET)/¯1+⍳NNX
[9]    →(0=⍴NIC)/S6

[10]   'THE NUMBER OF SOLUTIONS IS ZERO UNLESS THE
       FOLLOWING INPUT CONFIGURATIONS ARE FORBIDDEN: ';
       NIC
[11]  S6:DIS←(NNY,NNX)⍴((SET≠NNY)∧(SET≠0))
[12]   DIS←DSCR×DIS
[13]   DIS[1;]←(SET=0)∨(SET=NNY)∨DIS[1;]
[14]   SET←+⌿DIS
[15]   SOL←×/SET
[16]   MTX←DIS×MTX
[17]   'NUMBER OF SOLUTIONS AFTER CONSTRAINTS: SOL= ';SOL
[18]   →(SOL=0)/0
[19]   'DESIRED SOLUTION VECTOR:'
[20]   VEC←,⎕
[21]   LIM←⍴VEC
[22]   T←1
[23]  S4:FIX←1+SET⍳(VEC[T]-1)
[24]   RES←⍳0
[25]   V←1
[26]  S1:COL←(MTX[;V]>0)/MTX[;V]
[27]   RES←RES,COL[FIX[V]]
[28]   →(NNX≥V←V+1)/S1
[29]   MMR←(NY,NNX)⍴0
[30]   RES←RES-1
[31]   V←1
[32]  S2:MMR[;V]←V×(NY⍴2)⊤RES[V]
[33]   →(NNX≥V←V+1)/S2
[34]   MMR←⊖MMR
[35]   'SOLUTION NUMBER: ';VEC[T]
[36]   V←1
[37]  S3:'(Z ';V;') → ';FX[V;]←(0<MMR[V;])+2×(¯1+⍳NNX)∈
       (NIC,DON);'       [';¯1+((0<MMR[V;])/MMR[V;]);
       '] ∪ (';DON,NIC;')'
[38]   →(NY≥V←V+1)/S3
[39]   →(LIM≥T←T+1)/S4
     ∇

     ∇ SPACE
[1]    CM←1
[2]    'NUMBER OF Y-VARIABLES:'
[3]    NNY←2*NY←⎕
[4]    'SYMBOLS FOR Y-VARIABLES: (Y K), (X K); K= ';⍳NY
[5]    'NY= ';NY
[6]    'XY-SPACE SYMBOL:  F[Y;X]'
[7]    XX←⊖(BINX,NY⍴2)⊤(¯1+⍳NNF←2*NX+NY)
[8]    F←NNF⍴1
[9]    EQA←1
[10]   'CALL: EQUATION'
     ∇
```

```
    ∇ TABLE
[1]   ⍝ TRUTH TABLE FORMATTING
[2]    EQA←0
[3]    ' NUMBER OF FUNCTIONS:'
[4]    NF←NY←⎕
[5]    'TABLE IS READY FOR FUNCTIONS  (Z K)  WITH K =';⍳NF
[6]    BINF←NF⍴2
[7]    FX←(NF,NNX)⍴0
[8]    TR←TS←0
[9]    F←F←⍳0
[10]   'CALL OFFERINGS: FTRUE, FFALSE, FLIST'
[11]   TEX1←'(ANY LOGICAL APL-MEANINGFUL RELATION WITH
       VARIABLES  (X J), (~X J))'
    ∇


    ∇ R←X J
[1]    R←XX[J;]
    ∇


    ∇ R←Y K
[1]    R←XX[NX+K;]
    ∇


    ∇ R←Z L
[1]    R←FX[L;]
    ∇


    ∇ R←~X J
[1]    R←~XX[J;]
    ∇


    ∇ R←~Y K
[1]    R←~XX[NX+K;]
    ∇


    ∇ R←~Z L
[1]    R←(0=,FX[L;])+2×(2=,FX[L;])
    ∇


     ∇ ΔFFALSE
[1]    'SUFFICIENT CONDITION OF  F  MUST BE WRITTEN IN
       THE FORM:'
[2]    '~F←(LOGICAL RELATION IN  X-VARIABLES)'
[3]    'AND THE EXECUTION OF THIS INSTRUCTION MUST BE
       FOLLOWED'
[4]    'EITHER BY:  FSTOR K'
[5]    'OR BY:  FFALSE:'
     ∇
```

```
    ∇ ΔFTRUE
[1]   'SUFFICIENT CONDITION OF  F  MUST BE WRITTEN IN THE'
[2]   'PRESCRIBED FORM:'
[3]   'F←(LOGICAL RELATION IN  X-VARIABLES)'
[4]   'AND THE EXECUTION OF THIS INSTRUCTION MUST BE
      FOLLOWED'
[5]   'BY ONE OF THE PRESCRIBED CALLS:'
[6]   'EITHER:  FSTOR K   OR:  FFALSE'
    ∇
```

```
    ∇ ΔSYSTEM
[1]   'SOLVES FOLLOWING PROBLEMS:'
[2]   'TRUTH TABLE GENERATION OR EXISTENCE FUNCTION OF
      THE SYSTEM '
[3]   'TO GENERATE THE TABLE START WITH  LOGIC  AND
      CONTINUE'
[4]   'WITH   TABLE.'
[5]   'TO GENERATE EXISTENCE FUNCTION START WITH  LOGIC'
[6]   'AND THEN CALL  SPACE.'
    ∇
```

```
    ∇ ΔTABLE
[1]   'GENERATES TRUTH TABLE IN TWO WAYS:'
[2]   'CALL:  FTRUE OR FFALSE'
[3]   'TO DEFINE BOOLEAN FUNCTIONS BY ALGEBRAIC MEANS.'
[4]   'CALL:  FLIST'
[5]   'TO DEFINE FUNCTIONS BY DECIMAL EQUIVALENTS:'
    ∇
```